Ambiguity Aversion

I. An Experiment

Suppose there are 90 balls inside a box. It is known that 30 balls are red. The other 60 balls are either blue or green, but the exact number of each type is unknown. Suppose I am going to draw a ball out of the box. Consider the following two bets,

Bet A – you get $100 if the ball I draw is red, $0 otherwise.

Bet B – you get $100 if the ball I draw is blue, $0 otherwise.

Which bet would you choose?

Now consider these two bets,

Bet C – you get $100 if the ball I draw is either red or green, $0 otherwise.

Bet D – you get $100 if the ball I draw is either blue or green, $0 otherwise.

Which bet would you choose?

Are your choices rational according to expected utility?

This experiment is called the *Ellsberg Paradox*.

II. Risk and Uncertainty

To facilitate the study of ambiguity aversion, we make a distinction between risk and uncertainty.

Risk	–	Known probabilities
Uncertainty	–	Unknown probabilities

The natural question is then how to we evaluate situations where the probabilities of events are unknown.

III. Approach I: Maximin Utility

One way of thinking is to assume that the decision maker maximizes the utility from the worst scenario. In some sense, the DM tries to "cover all her bases". Mathematically, let X represents the bets the DM can choose from and S all possible combinations of probabilities, then the utility function can be represented by

$$\max_{x \in X} \left\{ \min_{s \in S} E[u(x)|s] \right\}$$

Looks kind of daunting? Applying it to our experiment at the beginning might clear things up.

More generally, we can allow the DM to have a utility that ranges from completely optimistic to completely pessimistic,

$$\max_{x \in X} \{\alpha \min_{s \in S} E[u(x)|s] + (1-\alpha) \max_{s' \in S} E[u(x)|s']\}$$

Question: What value of α would make the DM indifferent between all three colors of balls?

IV. Approach II: Subjective Utility

Another way to look at the problem is to assume that the DM assigns *subjective probabilities* to each possible outcome, which do not sum up to one. Once again let us apply this to the experiment.

In general, when a DM is pessimistic she chooses a subjective probability that gives her the lowest utility, whereas if she is optimistic she chooses a subjective probability that gives her the highest utility.

V. Application in Finance

If you have been following news on the current financial crisis, one of the issues facing financial firms is that markets for many fixed-income securities have "frozen"—there is no buyer, even if potential sellers are willing to decrease prices. Why might this happen? Ambiguity aversion offers a potential explanation.

Suppose there is only one risky asset in the world, and there are two possible outcomes—*H* and *L*, with subjective probabilities π_H and π_L respectively. If the outcome is *H*, stock price is p_H. If the outcome is *L*, stock price is p_L. The current stock price is p.